Assignment 1

LAURA MAYORAL
Instituto de Análisis Económico and Barcelona GSE

INSEAD, Spring 2018

Instructions: Please submit the problem set by May 18th. You can submit it electronically (at least the codes) to mayoralaura@gmail.com)

1. PROBLEMS

1. Consider the process $x_t = \phi x_{t-1} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise.

a. State the conditions that you need to impose on ϕ so that the inverse of the polynomial $\Phi(z)$=$(1-\phi z)$ exists.

b. Find $C(z)$, the inverse of $(1-\phi z)$.

c. Use your result in b. to find a representation of x_t in terms of past values of ε_t.

2. Compute the mean, the variance and the autocorrelations (up to lag 10) of the following processes. According to your results state whether they are covariance-stationary or not. In all cases $\{\varepsilon_t\}$ denotes a white noise sequence with variance equal to 1.

a. $x_t = 2 + \varepsilon_t + 2\varepsilon_{t-1}$

b. $y_t = 3 + 0.5y_{t-1} + \varepsilon_t$

c. $w_t = w_{t-1} + \varepsilon_t$ if t>0 and w_t=0 if t≤0.

*(Hints: To solve c, you can use backward substitution to express w_t in terms of ε_t).

3. Are the processes in exercise 2 strictly stationary? State the condition on ε_t that you could impose to obtain a sufficient condition so that (some of those processes) are in fact strictly stationary.

4. Are the processes in exercise 2 ergodic for the mean? and ergodic for second moments? Explain the implications of your answer in the estimation of the corresponding first and second moments.

5. Invertibility. Consider the following processes. ε_t is a white noise process in all cases.

a. $x_t = \varepsilon_t + 1.4\varepsilon_{t-1}$

b. $y_t = \varepsilon_t - \varepsilon_{t-1}$

c. $z_t = (1 - 1.5L)(1 - 0.2)\varepsilon_t$.

d. (1-0.3L)$w_t = (1 - 0.3L)(1 - 3/2L)(1 + 5/4L)\varepsilon_t$

Are these process invertible? Is it possible to find an alternative representation of these processes such that 1) it has the same correlation structure and 2) is invertible? Justify your answers.

(Hint: check Hamilton –Invertibility– if you have doubts).

6. Write a MATLAB function that allows to estimate by OLS an AR(p) process. The inputs of the function should be 1) the data, 2) the order of the AR. The output should be the estimated coefficients and their standard deviation. Note: You should include a constant in the regression.

7. From FREDII data base (`http://research.stlouisfed.org/fred2/`) download the series GDPC1 (quarterly US real GDP). Transform the series in growth rates. Suppose that real GDP growth rates follow an AR(1): $y_t = c + \phi y_{t-1} + \epsilon_t$, where $\{\epsilon_t\}$ is a white noise process.

a. Plot the series and describe its main patterns.

b. Estimate and plot the first 10 autocorrelations.

c. Estimate and plot the first 10 partial autocorrelations

d. Using the function you created in Exercise 6, estimate the parameters c and ϕ by OLS.

e. Now suppose that the series follows an AR(2) process. Estimate the parameters by OLS (the constant and the two AR coefficients).

f. Obtain the roots of the AR polynomial estimated in e).

2. Computer practice

(Note: You don't have to deliver this section)

Getting started with MATLAB

This section proposes very simple exercises using MATLAB. If you are not a MATLAB user yet, you can first watch the videos "Matlab Overview", "Getting Started" and "Writing a MATLAB Program" that you'll be able to find in Mathworks's website:

http://es.mathworks.com/products/matlab/videos.html

You are asked to write a .m file (i.e., an executable matlab file) that is able to do the following.

I. Working with matrices.

i)

- Create the matrices A=(1 2;3 7), B=(3 4)? and C=(2 1; 3 4);
- Compute: A+C; 3A; A^2 A^{-1}; $|A|$ and the eigenvalues of A.

- Compute a vector that contains the sum of the first row of A and B and a vector that contains the sum of the first colum of A and the second colum of C.
- Compute A*B' and A.*C (where the former is matrix multiplication and the latter is element by element multiplication)

ii. Create a matrix of ones of dimension (10, 20); create an identity matrix of the same dimension and a matrix of zeros of dimension (30,1);

iii. Create the following matrices of random numbers of dimension (200, 3) generated according the following distributions:

- D=N(0; 1).
- E=N(10,20)

Compute the sample mean and the standard deviation of the **columns** of these matrices and check that they are close to their corresponding theoretical values. Explain why is this case.

iv. Selecting elements in matrices.

- Select the element (1,2) of D and set it equal to 7.
- Select the first column of E and replace it by the first column of G
- Select the first column of F and replace by another vector given by (1,2,...200)

II. Writing functions.

Now, you are asked to write simple matlab *functions*. A matlab function is a .m file that will work as a built-in Matlab function (i.e., given an input the function will produced an output). You can type *help function* in the Matlab command window or this video for help `https://www.youtube.com/watch?v=Ii4tTkSk8As`.

i) Write a function that for a vector of data delivers two outputs: the mean and the standard deviation.

ii) Write a function such that for a vector of data computes the autocorrelation function up to lag k and plots this function (you can use the command *bar* to do so). The inputs to the function are the data and k whereas the output of the function is a vector containing the autocorrelation function up to lag k.

iii) Write a function that for a vector of data u_t generates $y_t = \phi y_{t-1} + u_t$. Thus, the inputs to the function are the data and ϕ, while the output is a vector of data generated by applying an AR(1) filter.

Good luck!!