# Assignment 3

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## 1. PROBLEMS

1. Impulse Response functions (IRFs). Read chapter 5.3. in Cochrane's book.

i) Explain the concept of IRF.

ii) Compute analitically the IRF of the following processes. $\varepsilon$ is a white noise process in all cases.

a. $(1-L)y_t = \phi_1(1-L)y_{t-1} + \varepsilon_t$

b. $x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \varepsilon_t$

c. $(1-L)z_t = \varepsilon_t + \theta\varepsilon_{t-1}$

d. $w_t = \alpha + \beta t + \varepsilon_t$

e. $v_t = \beta + v_{t-1} + \varepsilon_t$

c) Compute the IRF of the processes $y_t$ and $z_t$ defined in a. and c. above.

d) Compare the IRFs of processes in d and e. Describe how the sample ACF and the plot of the data of such processes would look like and then discuss on the differences between the IRFs.

e) Provide a formula for obtaining the IRF for a general AR(p) process. Hint: this formula relates IRF(h) to previous values of the IRF, and the AR coefficients.

2. ARCH models. Let $\{\varepsilon_t\}$ be an ARCH(2) process.

i) What are the characteristics of financial and macroeconomic data that ARCH models aim/can capture?

ii) Write down the first and second moments (ACF) of $\varepsilon_t$. Is $\varepsilon_t$ stationary? Clearly justify your answer.

iii) What condition(s) on the coefficients of the ARCH model you have to impose to ensure that $\varepsilon_t^2$ is stationary?

iv) Assuming stationarity of $\varepsilon_t^2$, write down the first and second moments of $\varepsilon_t^2$.

v) You have fitted a model to the process $y_t$ and the residuals seem to be white noise. Now you would like to test whether there is an ARCH effect in the innovations. In order to do this, you fit an AR(4) model to $e_t^2$,where $\{e_t\}$ are the residuals. You obtain that $\text{TR}_e^2 = 120$ where $\text{R}^2$ is the $\text{R}^2$ statistic associated to the AR regression. What would be your conclusion?

3. a) Consider the model

$$(1-L)^d y_t = \alpha + u_t,$$

where $u_t$ is a stationary process and $\alpha \neq 0$.

a) Discuss the role of $\alpha$ (what type of deterministic component it implies) for each of the possible values of $d = \{0, 1, 2\}$.

b) Show that if $y_t = a + \phi y_{t-1} + u_t$, where $u_t$ is an $AR(p)$ process, then $y_t$ can also be written as $\Delta y_t = \alpha^* + \phi_0^* y_{t-1} + \sum_{i=1}^{p} \phi_i^* \Delta y_{t-i} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise sequence.

c) Specify the values of $\alpha^*$ and $\phi_i^*$ in terms of the original parameters of the model.

d) Describe how you could test for a unit root in $y_t$, with $\alpha \neq 0$ and $u_t$ is an $AR(p)$ process. In particular, describe

i) the null and the alternative hypotheses

ii) the regression equation that you will estimate

$iii)$ different ways of controlling for the correlation in $u_t$

iv) what are the relevant critical values that you should use (and what the underlying assumptions that you are adopting to use these critical values) and in which cases you will reject the unit root hypothesis.

e) Consider the case where $u_t$ is an invertible and stationary process (but not necessarily an AR(p)). Describe how you could formulate a testing equation similar to that in b). (Hint: since $u_t$ is stationary and invertible, it has an AR($\infty$) representation). Discuss how one can test for a unit root in this more general case using the formulation that you just found. ( see Said and Dickey, Biometrika 1984 or Hamilton for details).

## 2. Computer practice

4. Consider the International Macro data in Stock and Watson. Find the appropriate transformations that are able to achieve stationarity for three of the processes in this dataset. Use unit root tests in order to do that.

5. Power of unit root tests

In this exercise you are asked to evaluate the power of u.r. tests. Generate R=1000 replications of an AR(1), $y_t = \phi y_{t-1} + \varepsilon_t$, $\varepsilon_t$ is $iidN(0,1)$ for $\phi = \{0.8, 0.90, 0.97\}$ and T= 250. Compute the DF regression, cases II and IV: $y_t = \beta_0 + \varphi y_{t-1} + \varepsilon_t$ and $\text{y}_t = \beta_0 + \beta_1 t + \varphi y_{t-1} + \varepsilon_t$. For a particular value of $\alpha$ (where $\alpha$ is the size of the test), for instance, 5%, construct a table that reports the power of this test (i.e., % of rejections of the null hypothesis of a unit root) of the test for each of the cases considered (different values of $\phi$ and regression models). While summarizing your conclusions please answer the following questions:

What are the appropriate set of critical values in this case? How does power change as $\phi$ approaches 1? why do you think this happens?

6. IRFs.

a. Write a MATLAB function that generates the IRF for an AR(1) process.

b. Try to write a function that generates the IRF for any AR(p) process. The answer to the question 1.e. above will help you in doing this.