Assignment 2

LAURA MAYORAL
Instituto de Análisis Económico and Barcelona GSE

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1. PROBLEMS

1. Show that a stationary AR(1) process is ergodic for first and second moments. Discuss what are the implications of this fact on the estimation of sample moments.

2. Consider the following ARMA (or ARIMA) processes. ε_t is a white noise process in all cases.

a. $(1-L)x_t = \mu + \varepsilon_t - 0.95\varepsilon_{t-1}$

b. a. $(1-L)y_t = \varepsilon_t - \varepsilon_{t-1}$

c. $z_t = (1-0.5L)(1-L)\varepsilon_t$

d. (1-0.6L)(1-0.4L)(1-0.2L)v$_t = (1-1.5L)(1-0.2L)\varepsilon_t$

e. (1-L)w$_t = (1-1/2L)(1+5/4L)\varepsilon_t$

i) State the orders p and q of the ARMA (or the ARIMA) processes above.

ii) Can the Wold theorem be applied on the processes? are these processes invertible? Clearly justify your answer.

3. You are interested in estimating ϕ in the following regression model: $y_t = \phi y_{t-1} + \varepsilon_t$. Estimation is carried out using OLS. Discuss the asymptotic properties of $\hat{\phi}_{ols}$ in the following cases.

i. $\{\varepsilon_t\}$ is $iN(0, \sigma^2)$

ii. $\{\varepsilon_t\}$ is white noise.

iii. $\varepsilon_t = \delta\varepsilon_{t-1} + v_t$, where v_t is white noise.

Discuss the assumptions you need to impose on ϕ to obtain these asymptotic properties.

4. A research wants to test hypothesis on ϕ, the AR(1) parameter from the model in exercise 3.i) . To that effect she constructs the t-statistic associated to the hypothesis $H_0 : \phi = 0$ and $H_0 : \phi = 1$. In both cases, her decision rule is as follows: she rejects the corresponding null hypothesis if the absolute value of the t-test is larger that the corresponding critical value obtained from the N(0,1). Discuss whether this procedure is right.

2. Computer practice

5. Using only visual tools (time series plots, ACFs and PACFs of the original data and possibly its first differences) try to identify the processes contained in the file PS2_data_ex5. (Note: It is an excel file).

6. Select three macroeconomic time series processes and following the steps we have seen in class identify, estimate and check the validity of the models you propose. You can use the Stock and Watson dataset (in the sample data in Gretl) but if you prefer you can also use other variables (check the course web for links).

Notes: use data without a seasonal component (either because the data is annual or because this component has been removed), since we haven't studied yet how to deal with this. If the data seems to have a non-constant mean, difference the process until you obtain something with well-behaved ACF and model the resulting (hopefully!) stationary processes.

7. Monte Carlo simulations using MATLAB.

In the following, you are asked to carry out a small Monte-Carlo simulation to check the properties of OLS estimators in autoregressions. To become more familiar with Monte Carlo methods, please read section 8.17 in Hansen's econometrics book (see the course's webpage). You can search some info in the web (e.g., http://en.wikipedia.org/wiki/Monte_Carlo_method).

The simulation goes as follows: generate 1000 processes of the form: $y_t = \phi y_{t-1} + \varepsilon_t$, for t=1,...T, where $\varepsilon_t \sim i.N\left(0,1\right), \phi = \{0.3\}$ and sample size T=100.

For each replication, estimate ϕ by OLS and save its value. Thus, you will have 1000 $\hat{\phi}'s$. Remember the OLS estimator of ϕ is simply given by

$$\hat{\phi} = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}.$$

Plot a histogram of the values that you have obtained. This histogram approximates the finite sample distribution of $\hat{\phi}$. Repeat the same exercise for other values of $\phi = \{0.8, 0.98\}$ and another sample size $T = \{1000\}$.

Compute a table that contains the bias and the mean square error –MSE–(that is, the variance +(the bias)^2) associated to $\hat{\phi}$ for each value of ϕ and each sample size. Compare the histograms that you have obtained and comment on these results: Does the estimator of $\hat{\phi}$ seem to be consistent? and asymptotically normal? What happens with the bias, the MSE and the approximation to the normal distribution when ϕ approaches 1?). To answer this last question, it would be also useful to plot the corresponding qqplots of the data.