Assignment 1

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1. PROBLEMS

1. Consider the process $x_t = \phi x_{t-1} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise.

a. State the conditions that you need to impose on ϕ so that the inverse of the polynomial $\Phi(z)$=$(1 - \phi z)$ exists.

b. Find $C(z)$, the inverse of $(1 - \phi z)$.

c. Show that C(z) is in fact the inverse of $\Phi(z)$ (that is, show that $C(z)\,\Phi(z) = 1$).

d. Use your result in b. to find a representation of x_t in terms of past values of ε_t.

2. Compute the mean, the variance and the autocovariance function of the following processes. According to your results state whether they are covariance-stationary or not. In all cases $\{\varepsilon_t\}$ denotes a white noise sequence with variance equal to 1.

a. $x_t = 2 + \varepsilon_t + 2\varepsilon_{t-1}$

b. $y_t = 3 + 0.5y_{t-1} + \varepsilon_t$

c. $z_t = 0.8z_{t-1} + u_t$, where $u_t = 0.3u_{t-1} + \varepsilon_t$.

d. $w_t = w_{t-1} + \varepsilon_t$ if t>0 and w_t=0 if t≤0.

e. $v_t = e_t$ if t>0 and v_t=0 if t≤0.

*(Hints: in order to solve b., invert the AR(1) polynomial as you did in Exercise 1. For c., write z_t as an AR(2) process. To solve d, you can use backward substitution to express w_t in terms of ε_t).

3. Are the processes in exercise 2 strictly stationary? State the condition on ε_t that you could impose to obtain that (some of those processes) are in fact strictly stationary.

4. Are the processes in exercise 2 ergodic for the mean? and ergodic for second moments? Explain the implications of your answer in the estimation of the corresponding first and second moments.

5. Show that if $\gamma(.)$ is the autocovariance function of a stationary process, then it verifies

$$\begin{aligned} i)\ \gamma(0) &\geq 0 \\ ii)\ |\gamma(h)| &\leq \gamma(0) \text{ for all } h \in \mathbb{Z} \\ iii)\ \gamma(-h) &= \gamma(h) \text{ for all } h \in \mathbb{Z} \end{aligned}$$

Hint: use the Cauchy-Schwarz inequality to show ii).

6. Invertibility. Consider the following processes. ε_t is a white noise process in all cases.

a. $x_t = \varepsilon_t + 1.4\varepsilon_{t-1}$

b. $y_t = \varepsilon_t - \varepsilon_{t-1}$

c. $z_t = (1 - 1.5L)(1 - 0.2)\varepsilon_t$.

d. $(1\text{-}0.3L)w_t = (1 - 0.3L)(1 - 3/2L)(1 + 5/4L)\varepsilon_t$

Are these process invertible? Is it possible to find an alternative representation of these processes such that 1) it has the same correlation structure and 2) is invertible? Justify your answers.

(Hint: check Hamilton –Invertibility– if you have doubts).

7. Next we are going to plot some variables together with the autocorrelation and partial autocorrelation function. You can easily do this in Gretl. For doing that, you need to first download and install the program.

Then, consider the International Macro data in Stock and Watson (you can find these data already loaded in Gretl, see File/open file/ Sample file/sw$_c h12$).

a. Choose three variables. Plot each of then (in logs) as well as their rate of growth*. Describe the main patterns that you observe in these series.

b. Plot the autocorrelation function of the log of the original data as well as that of their rate of growth. Describe the main patterns that you observe in these graphs. Do the data look stationary? and the rates of growth?

c. Plot the partial correlation function of the rates of growth. Discuss whether any of these series look like an AR or an MA process.

(*You can compute this as the first difference of the log of the original data.)

2. Computer practice

Getting started with MATLAB

This section proposes very simple exercises using MATLAB. If you are not a MATLAB user yet, you can first watch the videos “Matlab Overview”, ”Getting Started” and ”Writing a MATLAB Program” that you’ll be able to find in Mathworks’s website:

http://es.mathworks.com/products/matlab/videos.html

You are asked to write you a .m file (i.e., an executable matlab file) that is able to do the following.

I. Working with matrices.

i)

- Create the matrices A=(1 2;3 7), B=(3 4)? and C=(2 1; 3 4);
- Compute: A+C; 3A; A^2 A^{-1}; $|A|$ and the eigenvalues of A.
- Compute a vector that contains the sum of the first row of A and B and a vector that contains the sum of the first colum of A and the second colum of C.
- Compute A*B’ and A.*C (where the former is matrix multiplication and the latter is element by element multiplication)

ii. Create a matrix of ones of dimension (10, 20); create an identity matrix of the same dimension and a matrix of zeros of dimension (30,1);

iii. Create the following matrices of random numbers of dimension (200, 3) generated according the following distributions:

- D=N(0; 1).
- E=N(10,20)
- F=U(0,1)
- G=U(10,30)
- H=χ^2_3
- I=t_4

Compute the sample mean and the standard deviation of the **columns** of these matrices and check that they are close to their corresponding theoretical values. Explain why is this case.

iv. Selecting elements in matrices.

- Select the element (1,2) of D and set it equal to 7.
- Select the first column of E and replace it by the first column of G
- Select the first column of F and replace by another vector given by (1,2,...200)

II. Writing functions.

Now, you are asked to write simple matlab *functions*. A matlab function is a .m file that will work as a built-in Matlab function (i.e., given an input the function will produced an output). You can type *help function* in the Matlab command window or this video for help https://www.youtube.com/watch?v=Ii4tTkSk8As.

i) Write a function that for a vector of data delivers two outputs: the mean and the standard deviation.

ii) Write a function such that for a vector of data computes the autocorrelation function up to lag k and plots this function (you can use the command *bar* to do so). The inputs to the function are the data and k whereas the output of the function is a vector containing the autocorrelation function up to lag k.

iii) Write a function that for a vector of data u_t generates $y_t = \phi y_{t-1} + u_t$. Thus, the inputs to the function are the data and ϕ, while the output is a vector of data generated by applying an AR(1) filter.

Good luck!!