Assignment 2

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Instructions: The deadline is March 13. Please submit your answers individually electronically or in paper to Nicolo Maffei, nicolo.maffei@barcelonagse.eu.

1. PROBLEMS

1. Show that a stationary AR(1) process is ergodic for first and second moments. Discuss the implications of this fact on the estimation of the first and second population moments.

2. Consider the process X_t and assume it is weakly stationary with $E(X_t) = 3$.

i) Write a model for this process and describe its main characteristics.

ii) Compute its autocorrelation function.

iii) Can you write X_t as an AR(∞) process? State the conditions you would need to impose so that your answer is affirmative.

3. Consider the following processes. The sequence $\{\varepsilon_t\}$ is white noise in all cases.

a. $x_t = \mu + \varepsilon_t - \varepsilon_{t-1}$

b. $y_t = y_{t-1} + \varepsilon_t - 0.95\varepsilon_{t-1}$

c. $z_t = z_{t-1} + (1 - 0.5L)(1 - L)\varepsilon_t$

d. $(1 - L + 0.24L^2)v_t = (1 - 0.6L)\varepsilon_t$

i) State which of the above processes are ARMA(p,q) and for those processes state the values of p and q. (Note: Remember that ARMA processes are stationary. Note for c.: if a process has a unit root both in its MA and AR components, you can simplify this term from both sides of the equation).

ii) Whenever possible write the processes above as AR processes.

4. You are interested in estimating ϕ in the following regression model: $y_t = \phi y_{t-1} + \varepsilon_t$. Estimation is carried out using OLS.

a) Assuming that $|\phi| < 1$, discuss the asymptotic properties of $\hat{\phi}_{ols}$ in the following cases.

i. $\{\varepsilon_t\}$ is $iN(0, \sigma^2)$

ii. $\varepsilon_t = \delta\varepsilon_{t-1} + v_t$, where v_t is a martingale difference sequence.

b) Assuming that ε_t is defined as in ii), rewrite y_t as an AR(2) process with white noise residuals (v_t) and find the values of the AR coefficients as functions of ϕ and δ. Suppose that you estimate the resulting process by OLS. Is the estimator consistent in this case?

c) Consider now $y_t = \beta x_t + \varepsilon_t$ where ε_t is defined as in ii). Assuming that x_t and y_t are stationary and $cov(x_t, \varepsilon_t) = 0$, is the OLS estimator of β consistent in this case?

5. A research wants to test hypotheses on ϕ, the AR(1) parameter from the model in exercise 3.i) . To that effect she constructs the t-statistic associated to the hypothesis $H_0 : \phi = 0$ and $H_0 : \phi = 1$. In both cases, her decision rule is as follows: she rejects the corresponding null hypothesis if the absolute value of the t-test is larger that the corresponding critical value obtained from the N(0,1). Discuss whether this procedure is right.

2. COMPUTER PRACTICE

6. Using MATLAB, construct a function that for a given data, i) selects the order k using the General-to-Specific selection method corresponding to an AR specification and ii) estimates the corresponding AR(k) process using OLS. The input of the function should be your data and the maximum lag for the AR. The output should be k and the estimates and variances of the estimates.

7. Monte Carlo simulations using MATLAB.

In the following, you are asked to carry out a small Monte-Carlo simulation to check the properties of OLS estimators in autoregressions. To become more familiar with Monte Carlo methods, please read section 8.17 in Hansen's econometrics book (see the course's webpage). You can search some info in the web (e.g., http://en.wikipedia.org/wiki/Monte_Carlo_method).

The simulation goes as follows: generate 1000 processes of the form: $y_t = \phi y_{t-1} + \varepsilon_t$, for t=1,...T, where $\varepsilon_t \sim i.N(0,1), \phi = \{0.3\}$ and sample size T=100.

For each replication, estimate ϕ by OLS and save its value. Thus, you will have 1000 $\hat{\phi}'s$. Remember the OLS estimator of ϕ is simply given by

$$\hat{\phi} = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}.$$

Plot a histogram of the values that you have obtained. This histogram approximates the finite sample distribution of $\hat{\phi}$. Repeat the same exercise for other values of $\phi = \{0.8, 0.98\}$ and another sample size $T = \{1000\}$.

Compute a table that contains the bias and the mean square error –MSE–(that is, the variance +(the bias)^2) associated to $\hat{\phi}$ for each value of ϕ and each sample size. Compare the histograms that you have obtained and comment on these results: Does the estimator of $\hat{\phi}$ seem to be consistent? and asymptotically normal? What happens with the bias, the MSE and the approximation to the normal distribution when ϕ approaches 1?). To answer this last question, it would be also useful to plot the corresponding qqplots of the data.