Problemset 1

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Instructions: The deadline is January 27th. You can submit your answers electronically or in paper to Nicolo Maffei, nicolo.maffei@barcelonagse.eu.

1. PROBLEMS

1. Consider the process $x_t = \phi x_{t-1} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise.

a. State the conditions that you need to impose on ϕ so that the inverse of the polynomial $\Phi(z)$=$(1-\phi z)$ exists.

b. Find $C(z)$, the inverse of $(1-\phi z)$.

c. Use your result in b. to find a representation of x_t in terms of past values of ε_t.

2. Compute the mean, the variance and the autocorrelation function (up to lag 10) of the following processes. According to your results state whether they are covariance-stationary or not. In all cases $\{\varepsilon_t\}$ denotes a white noise sequence with variance equal to 1.

a. $x_t = 2 + \varepsilon_t + 2\varepsilon_{t-1}$

b. $y_t = 3 + 0.5y_{t-1} + \varepsilon_t$

c. $z_t = 4 + \varepsilon_t$.

3. Are the processes in exercise 2 strictly stationary? State the condition on ε_t that you could impose to obtain a sufficient condition so that (some of those processes) are in fact strictly stationary.

4. Are the processes in exercise 2 ergodic for the mean? and ergodic for second moments? Explain the implications of your answer in the estimation of the corresponding first and second moments.

5. Invertibility. Consider the following processes. ε_t is a white noise process in all cases.

a. $x_t = \varepsilon_t + 1.4\varepsilon_{t-1}$

b. $y_t = \varepsilon_t - \varepsilon_{t-1}$

c. $z_t = (1-1.5L)(1-0.2)\varepsilon_t$.

d. (1-0.3L)$w_t = (1-0.3L)(1-3/2L)(1+5/4L)\varepsilon_t$

Are these process invertible? Is it possible to find an alternative representation of these processes such that 1) it has the same correlation structure and 2) is invertible? Justify your answers.

(Hint: check Hamilton –Invertibility– if you have doubts).

6. Write a MATLAB function that allows to estimate by OLS an AR(p) process, $y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_t$. The inputs of the function should be 1) the data, 2) the order of the AR. The output should be the estimated coefficients and their standard deviation. Note: You should include a constant in the regression.

7. From FREDII data base (`http://research.stlouisfed.org/fred2/`) download the series GDPC1 (quarterly US real GDP). Transform the series in growth rates. Suppose that real GDP growth rates follow an AR(1): $y_t = c + \phi y_{t-1} + \epsilon_t$, where $\{\epsilon_t\}$ is a white noise process.

a. Plot the series and describe its main patterns.

b. Estimate and plot the first 10 autocorrelations.

c. Estimate and plot the first 10 partial autocorrelations

d. Using the function you created in Exercise 6, estimate the parameters c and ϕ by OLS.

e. Now suppose that the series follows an AR(2) process. Estimate the parameters by OLS (the constant and the two AR coefficients).

f. Obtain the roots of the AR polynomial estimated in e).