Assignment 4

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Instructions: You need to submit this problem set by February 14th. Please submit it individually and electronically to this address: timeseries2018@gmail.com. As a subject in your email please add "Problem Set 4".

1. PROBLEMS

0. Read Luetkepohl (2011)'s VAR article on the course's website.

1. Consider the following n-dimensional VMA process

$$y_t = \varepsilon_t + C_1 \varepsilon_{t-1}$$

where ε_t is a WN$(0, \Omega)$.

(a) State the condition under which the process is invertible.

(b) Derive the variance of the process ($E(y_t y_t')$).

(c) Derive the autocorrelation function.

(d) Under the assumption that the process is invertible, obtain the VAR representation.

(e) Obtain a representation of the VMA above in terms of a new vector of shocks whose covariance matrix is the identity matrix.

(f) Assume y_t is a bivariate process with $C_1 = \begin{bmatrix} 1 & \theta \\ 0.5 & 0 \end{bmatrix}$, what are the values of θ for which y_t is invertible?

2. Consider the following VAR(2)

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \varepsilon_t$$

where y_t is a 2×1 vector and ε_t is a 2-dimensional white noise vector with covariance matrix Ω.

(a) State the condition under which (1) is stable (and therefore stationary).

(b) Write the companion form of the VAR(2).

(c) Assume the model is stable. Derive the Wold representation defining the coefficients of such representation in terms of the VAR parameter matrices.

2. COMPUTER PRACTICE

5. **Power of unit root tests**

In this exercise you are asked to evaluate the power of unit root tests. In order to do that, you can proceed as follows:

Generate R=1000 replications of an AR(1) process, $y_t = \phi y_{t-1} + \varepsilon_t$, where ε_t is $iidN(0,1)$, for different values of the AR coefficient, $\phi = \{0.8, 0.90, 0.97\}$ and T= 250. Compute the DF regression, cases II and IV: $y_t = \beta_0 + \varphi y_{t-1} + \varepsilon_t$ and $y_t = \beta_0 + \beta_1 t + \varphi y_{t-1} + \varepsilon_t$. For a particular value of α (where α is the size of the test), for instance, 5%, construct a table that reports the power of this test (i.e., % of rejections of the null hypothesis of a unit root) of the test for each of the cases considered (different values of ϕ and regression models). While summarizing your conclusions please answer the following questions:

What are the appropriate set of critical values in this case? How does the power of the test evolve as ϕ approaches 1? why do you think this happens?

6. **Interest rate spread and output growth in the US.** From the FRED database of the St. Louis Fed (http://research.stlouisfed.org/fred2/) down- load data for the 10Y government bond (mnemonic GS10), the federal funds rate (mnemonic FEDFUNDS) and the real GDP (mnemonic GDPC1).

(1) Create the quarterly spread as the quarterly average of the difference between the long and the short rate. Create the growth rate of real GDP.
(2) Are these variables stationary? use unit root tests to justify your answer.
(3) Using the growth rates of real GDP and the spread (use all the observations available), estimate a VAR(p). Choose p according to an IC.
(4) Compute and plot the impulse response functions of the Wold representation together with the 68% confidence bands obtained with the bootstrap method.
(5) Perform a test of Granger causality in both directions.
(6) Compute the MSE of the (out-of-sample) forecast of output growth one- and four- period ahead using data up to 1985:Q1 for the initial estimation and using the post- 1985:Q1 period for the forecast evaluation.