Assignment 3

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Instructions: You need to submit this problem set by February 7th. Please submit it individually and electronically to this address: timeseries2018@gmail.com.

1. PROBLEMS

1. Impulse Response functions (IRFs). Read chapter 5.3. in Cochrane's book.

i) Explain the concept of IRF.

ii) Compute analitically the IRF of the following processes. $\{\varepsilon_t\}$ is a white noise process in all cases. Note: In a. and c. compute the IRF of the first difference of the original processes y_t and z_t .

a. $(1-L)y_t = \phi_1(1-L)y_{t-1} + \varepsilon_t$

b. $x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \varepsilon_t$

c. $(1-L)z_t = \varepsilon_t + \theta\varepsilon_{t-1}$

d. $w_t = \alpha + \beta t + \varepsilon_t$

e. $v_t = \beta + v_{t-1} + \varepsilon_t$

iii) Compute the IRF of the processes y_t and z_t defined in a. and c. above.

iv) Provide a formula for obtaining the IRF for a general AR(p) process. Hint: this formula relates IRF(h) to previous values of the IRF, and the AR coefficients.

v) Describe how the sample ACF and the plot of the data of the processes in d. and e would look like. Compare the IRFs of those processes and explain what these differences mean in terms of the persistence of shocks in both models.

2. ARCH models. Read Engle's Nobel Lecture for more information on ARCH processes.

Let $\{\varepsilon_t\}$ be an ARCH(2) process.

i) What are the characteristics of financial and macroeconomic data that ARCH models aim/can capture?

ii) Write down the first and second moments (ACF) of ε_t. Is ε_t stationary? Clearly justify your answer.

iii) What condition(s) on the coefficients of the ARCH model you have to impose to ensure that ε_t^2 is stationary?

iv) Assuming stationarity of ε_t^2, write down the first and second moments of ε_t^2.

v) You have fitted a model to the process y_t and the residuals seem to be white noise. Now you would like to test whether there is an ARCH effect in the innovations. In order to do this, you fit an AR(4) model to e_t^2,where $\{e_t\}$ are the residuals. You obtain that $\text{TR}_e^2 = 27$ where R^2 is the R^2 statistic associated to the AR regression. What would be your conclusion?

3. a) Consider the model

$$(1 - L)^d \, y_t = \alpha + u_t,$$

where u_t is a stationary process and $\alpha \neq 0$.

a) Discuss the role of α (i.e., the type of deterministic component it implies) for each of the possible values of $d = \{0, 1, 2\}$.

b) Show that if $y_t = a + \phi y_{t-1} + u_t$, where u_t is an $AR\,(p)$ process, then y_t can also be written as $\Delta y_t = \alpha^* + \phi_0^* y_{t-1} + \sum_{i=1}^{p} \phi_i^* \Delta y_{t-i} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise sequence. (Check Hamilton if you need help).

c) Specify the values of α^* and ϕ_i^* in terms of the original parameters of the model.

d) Describe how you could test for a unit root in y_t, if you don't know the value of α and u_t is a white noise process.

In particular, describe

i) the null and the alternative hypotheses

ii) the regression equation that you will estimate

iii) what are the relevant critical values that you should use.

iv) Describe your decision rule if T=1000 (that is, the values of the t-test for which for which you would you reject the null hypothesis).

v) Suppose that you don't know any unit root theory and mistakingly use the critical values of the normal distribution. Suppose that you want to run a one-sided test of size 10% (i.e., $\alpha = 0.1$). Then, what would be the (approximate) size of your t-test if you use that critical value? (i.e., how often you would reject the null hypothesis when it is true for that critical value). Summarize your conclusions.

2. Computer practice

8. Monte Carlo simulation using Matlab.

In the following, you are asked to carry out a small Monte-Carlo simulation to check the properties of OLS estimators in autoregressions. To become more familiar with Monte Carlo methods, please read section 8.17 in Hansen's econometrics book (see the course's webpage). You can search some info in the web (e.g., http://en.wikipedia.org/wiki/Monte_Carlo_method).

You can follow the following steps:

(1) Generate R=1000 processes of the form: $y_t = \phi_0 y_{t-1} + \varepsilon_t$, for t=1,...T, where $\varepsilon_t \sim i.N(0,1)$, $\phi_0 = \{0.3\}$, and sample size T=400.
(2) For each replication r, estimate ϕ_0 by OLS and save the result.
(3) For each replication r, compute the t-test associated to the hypothesis $H_0 : \phi = \phi_0$ and save the result.
(4) Once you've computed this R times, plot a histogram of the values that you have obtained in 2). This histogram approximates the finite sample distribution of $\hat{\phi}$. Do the same for the values you've obtained in 3). This histogram approximates the finite sample distribution of the t-statistic.
(5) Repeat steps 1–4 for $\phi_0 = \{0.90\}$ and $\phi_0 = \{0.97\}$.

i) For each value of ϕ_0 compute the bias and the mean square error associated to $\hat{\phi}$ (that is, the variance +(the bias)^2, check `https://en.wikipedia.org/wiki/Mean\_squared\_error`). How do these quantities evolve as ϕ_0 approaches 1?

ii) Compare the histograms that you have obtained. Does the estimator of $\hat{\phi}$ seem to be consistent? and asymptotically normal? What happens with the approximation to the normal distribution when ϕ approaches 1?). To answer this last question, it would be also useful to plot the corresponding qqplots of the data.