Uncertainty, model selection and the PPP puzzle

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Abstract

This paper employs a Bayesian methodology to measure the uncertainty involved in the long-run holding of the purchasing power parity (PPP) and in the half-life (HL) of deviations from it. Our Bayesian approach combines estimates obtained from ARMA, ARFIMA and ARIMA specifications so that it is able to take into account model uncertainty. Exact posterior distributions corresponding to all the features of interest (fractional orders of integration, impulse responses, HLs, etc.) are derived. Our results suggest that model uncertainty is high, which implies that uncertainty estimates can be severely biased downwards when the former is not explicitly considered. In spite of that, support for the long-run holding of the PPP is substantial in most countries. However, the speed at which this happens seems to be much slower than what has been suggested, as shown by the fact that the probability of the HL lying in the 3 to 5 year interval is very small.

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1 Introduction

What is the persistence of deviations from purchasing power parity? In recent years, several authors have questioned the validity of the so-called 'Rogoff's consensus', i.e., the observation that half-lives (HLs) of deviations from parity usually fall in the range of 3 to 5 years (Murray and Papell, 2002, 2005, Rossi, 2005, Lopez et al., 2005, 2013). Using bias-corrected estimation methods in autoregressive (AR) processes, these authors obtain considerably higher point estimates. In addition, they underscore the high uncertainty involved in these estimates, as reflected by confidence intervals that are often too wide to attain unequivocal conclusions on whether or not HLs are consistent with purchasing power parity (PPP).

The goal of this paper is to probe deeper into the uncertainty involved in estimates of the persistence of deviations from PPP. Our approach has two distinctive characteristics. First, we use fractionally integrated (FI) processes to fit RER data (rather than AR models, as in the above-mentioned papers). This is motivated by recent literature (Crucini et al., 2005, 2010, Imbs et al., 2005, Mayoral and Gadea, 2011) that emphasizes the large degree of heterogeneity in the dynamics of sectoral RERs. If the sectors can be modeled as (heterogeneous) autoregressions, their aggregate is still AR but it contains an infinite number of terms (Lewbel, 1994). Thus, very long models have to be fitted to aggregate RERs to obtain well-behaved estimators (Mayoral, 2013). An alternative approach is to use FI models. As shown by Granger (1980), the aggregate of heterogeneous AR models can be approximated in a parsimonious way by a FI process.[1] This parsimony is potentially very beneficial since it can contribute to reducing the size of the estimators' confidence intervals.

Second, we adopt a Bayesian approach, similar to that employed by Koop et al. (1997).

[1] See Zaffaroni (2004) for a full characterization of the conditions needed for this approximation.

In this framework this approach has at least three advantages over the classical estimation. First, the method takes model uncertainty into account when making inferences about the relevant parameters, which is something that cannot be done in the classical estimation. In particular, it allows us to average across models rather than choosing just one of them. Second, it provides exact finite sample distributions for any feature of interest (e.g., the fractional differencing parameter, an impulse response or the HL). Thus, instead of presenting just a point estimate and the standard error associated with, say, the HL, we can plot the whole density of that quantity of interest. This might be important since, as shown by Koop et al. (1994), impulse responses can be multimodal, highly skewed and fat-tailed. Furthermore, this also allows us to compute interesting probabilities, such as the probability that the PPP holds, $P(d < 1)$ or the probability of the so-called Rogoff interval (HLs in the interval of 3 to 5 years), and perform small-sample tests of memory properties. Third, it has been argued that, although FI(d) models encompass the $I(1) - I(0)$ setup, in practice, it is very unlikely that an integer value of d is obtained when the true value is, in fact, an integer. As pointed out by Hauser et al. (1999), this could be one drawback of FI models, especially when they are used for measuring persistence. Nevertheless, the Bayesian approach adopted here allows us to attach a prior probability mass to integer values of d, $d = \{0, 1\}$ so that the ARMA/ARIMA and ARFIMA specifications are on an equal footing. This will be particularly useful in Section 4 when persistence measures are computed.

Our results can be summarized as follows. The empirical exercise opens with a classical analysis of a group of long-horizon RERs originally compiled by Taylor (2002). Using (large-sample based) tests and estimation methods, we show that there is strong support for fractional integration and the holding of the PPP (i.e., values of d smaller than 1). These findings are robust to considering alternative linear and nonlinear representations for RERs. The adoption of a Bayesian approach, however, allows us to uncover the fact that model uncertainty is considerably higher than what the classical analysis suggests. It is obtained that (pure) ARFIMA models receive, in general, large posterior probabilities (around 50% on average), which underscores the importance of sectoral heterogeneity in RER data. However, ARMA and ARIMA formulations also receive substantial posterior probability (27% and 23%, respectively). Thus, there is evidence in favor of fractional orders of integration, although not overwhelming. Furthermore, the posterior probability is quite scattered across the twenty-seven models considered, which shows the hazards of selecting just one. In spite of the large uncertainty, the holding of the PPP, implied by an order of integration smaller than 1, has considerable support in our data. We obtain an average posterior probability associated with $d < 1$ equal to 0.72. In contrast, if model uncertainty is not taken into account, this probability rises to 98%, a result that parallels the one obtained with classical methods. The difference between these two probabilities suggests that the measures of uncertainty typically provided in most papers may suffer from severe biases.

Our results deserve more comments. The speed of convergence to equilibrium appears to be very slow. The median values of the HL are quite heterogeneous across countries but, in general, they are larger than 10 years for more than half of the countries in the sample, confirming the very persistent character of deviations from equilibrium. Not surprisingly, the probability that the HL lies in the so-called Rogoff's interval (3 to 5 years) appears to be quite small (21% on average), and it is only larger than 50% for two countries (Italy and Sweden).

The results described above differ considerably from those reported in the original work of Taylor (2002), who obtains a mean HL of around 4 years, and are closer to those of Murray and Papell (2002, 2005), Lopez et al. (2005, 2013), Caporale et al. (2005) and

Rossi (2005).

In sum, our findings highlight the importance of taking model selection uncertainty into account when quantifying the uncertainty involved in the long-run holding of the PPP and in the HL of deviations from PPP. However, in spite of the high uncertainty, the data seems to be informative about several issues: the PPP is likely to hold in the long run for most of the countries considered in this paper, although the rate of reversion to parity appears to be extremely slow, with HLs that are considerably larger than 3-5 years.

This is not the first paper that considers FI models or Bayesian methods to analyze aggregate RERs but, to the best of our knowledge, it is the first that combines both. Diebold et al. (1991) and Cheung and Lai (1993) found evidence of fractional orders of integration smaller than 1 in long historical series of real exchange rates, while Cheung and Lai (2001) and Achy (2003) reported similar results in the recent floating rate period. Nevertheless, Baum et al. (1999) failed to reject the unit root hypothesis against the fractional integration alternative for the recent float. Along similar lines, Okimoto and Shimotsu (2010) investigated the possibility of a decline in persistence, measured by a decrease of the fractional degree of integration, in a group of post-Bretton Woods RERs. Although they find support for this conjecture for half of the countries considered in their study, this decline is not sufficient for PPP to hold.

Kilian and Zha (2002) adopt a Bayesian approach to quantify the uncertainty involved in the HL of deviations from PPP but their approach is quite different to ours. They propose a prior probability distribution for the HL based on the responses to a survey study in which economists were questioned about their subjective prior probabilities for the HL in the post-Bretton Woods era. They use long AR models (AR(12)) to fit the data and derive the posterior probability of the HL from this *consensus* of prior probabilities, finding substantial uncertainty about the HL. Also using Bayesian techniques, Lo and

Morley (2013) investigate exchange rate nonlinearities and the PPP persistence puzzle, finding support for nonlinearities and shorter speeds of reversion.

The remainder of the paper is organized as follows. Section 2 examines the stochastic properties of the aggregate process when sectoral heterogeneity is allowed and reviews the main properties of FI models. Sections 3 and 4 contain the empirical results. Section 3 reports the results of modeling RER data using both classical and Bayesian methods. Section 4 presents half-life estimates and provides measures of the uncertainty associated with the estimates while Section 5 concludes.

2 Sectoral heterogeneity and the modeling of the aggregate process

As noticed by Imbs et al. (2005), "*one recurrent conclusion in most of the existing work [on sectoral exchange rates] is heterogeneity, both across sectors and across countries.*" Indeed, evidence supporting this statement is ample (see, for instance, Crucini et al., 2005, Crucini and Shintani, 2008, Mayoral and Gadea, 2011, etc.).

What are the implications of sectoral heterogeneity on the modeling of aggregate RERs? Consider a simple model for q_{it}, the sectoral real exchange rate for sector i between the domestic country and the U.S. that allows for dynamic heterogeneity at the sectoral level,[2]

$$q_{it} = \beta_i + \lambda_i q_{it-1} + \nu_{it}, \; i = 1, ..., \; N, \; t = 1, ...T, \tag{1}$$

where N denotes the number of sectors, $q_{it} = p_{it} - p_{us,it} - e_t$, p_{it} and $p_{us,\, it}$ are the log of the price in sector i corresponding to the domestic country and the U.S., respectively, and e_t is the log of the nominal exchange rate, defined as domestic currency units per U.S.

[2] This specification is common in the analysis of sectoral exchange rates (see Imbs et al., 2005, Crucini and Shintani, 2008 and Gadea and Mayoral 2009, among others).

dollar. The aggregate real exchange rate is constructed as a (weighted) average of sectoral exchange rates,

$$Q_t = \sum_{i=1}^{N} \omega_i q_{it}, \tag{2}$$

where $\sum_{i=1}^{N} \omega_i = 1$, and ω_i are weights. Assuming that the coefficients β_i, λ_i and ρ_i are drawn from the distributions of the random variables β, λ and ρ, Lewbel (1994) shows that Q_t follows an AR(∞) process, given by

$$Q_t = A_0 + \sum_{k=1}^{\infty} A_k Q_{t-k} + u_t, \tag{3}$$

for some constants A_1, A_2,[3]... It follows that, in the presence of heterogeneity, the dynamics of the aggregate process become quite involved. Although consistent and asymptotically normal estimates of the model parameters can be obtaining by fitting (long) AR(K) models to Q_t (Berk, 1974), in practice, estimation is complicated by two problems. First, model selection in AR(∞) models can be a difficult task. As shown by Kuersteiner (2005), some of the most popular information criteria, such as the AIC and the BIC, tend to choose models that are not large enough to obtain consistent and asymptotically normally distributed estimates of the coefficients. This will obviously translate into unreliable persistence estimates. Second, given the usual shortness of macroeconomic series, fitting long autoregressions will most likely translate into wide confidence intervals for all the estimates, including persistence measures.

2.1 Heterogeneity and Fractional Integration

An alternative to using long autoregressions to model aggregate RERs in the presence of sectoral heterogeneity is to consider fractionally integrated (FI) processes. If sectoral RERs

[3] More specifically, $A_k = E(\alpha_k)$, where $\alpha_1 = \lambda$ and $\alpha_k = (\alpha_{k-1} - A_{k-1})\lambda$ for $k > 1$, and $u_t = E(\nu_{it})$.

are defined as in (1), Q_t can be characterized as a FI process under certain conditions. More specifically, building on previous results by Robinson (1978) and Granger (1980), Zaffaroni (2004) shows that the stochastic properties of Q_t when $N \to \infty$ depend on the behavior of the distribution of λ, the heterogeneous AR coefficient, around 1. Let the support of λ be $[0,\gamma]$, where negative values of λ are excluded for simplicity's sake. If $\gamma < 1$, the aggregate process behaves as a stationary $I(0)$ process. If $\gamma = 1$ and the $\mathrm{P}(\lambda = 1) > 0$, then Q_t contains an (exact) unit root. An interesting intermediate case arises whenever the distribution of λ is absolutely continuous in the interval $[0, 1)$. To characterize the properties of Q_t in this case, Zaffaroni (2004) uses a semiparametric characterization of the density of λ, $f(\lambda)$, around unity,

$$f(\lambda) \sim c_d (1-\lambda)^{-d}, \text{ as } \lambda \to 1, \; 0 < c_d < \infty, \tag{4}$$

where '$\sim$' stands for asymptotic equivalence, c_d is a constant and d is a real number.[4] If the distribution of λ verifies this condition, then Q_t converges to a fractionally integrated process of order d, given by

$$(1-L)^d (Q_t - \mu) = X_t, \tag{5}$$

where X_t is an $I(0)$ process and

$$\begin{aligned} (1-L)^\delta &= \sum_{i=0}^{\infty} \pi_i(\delta) L^i, & (6)\\ \pi_i(\delta) &= \Gamma(i-\delta) / (\Gamma(-\delta)\Gamma(i+1)), & (7) \end{aligned}$$

and $\Gamma(.)$ denotes the gamma function.

[4]This semiparametric specification of the cross-sectional distribution of λ leaves the behavior of the density function of this parameter completely unspecified for any given interval $[0, \varsigma]$ with $\varsigma < 1$. A particular case of this general family of distributions is the *Beta* distribution.

Since heterogeneity is widely documented in sectoral RER data, FI models provide a plausible parametrization for aggregate exchange rates. These processes present some potential advantages in this framework. First, it is well known that estimates of AR models can be severely biased if there is a lot of persistence in the data. In fact, several authors have shown that these biases are substantial in RERs (Murray and Papell, 2002, Lopez et al., 2013, etc.), so finite-sample corrections are needed to obtain reliable persistence measures. Persistence in FI models is mainly captured by d, the memory parameter, (rather than by the AR coefficients) and, in general, its estimators do not suffer from systematic finite-sample biases. Therefore, there is no need to apply bias-correcting mechanisms. Second, in the presence of sectoral heterogeneity, FI models may provide more parsimonious specifications than that implied by (3). This is because, under the previous assumptions, the memory parameter d is able to capture the long-run pattern of correlations that the aggregation of heterogeneous processes produces more efficiently than AR terms.

2.2 ARFIMA models and persistence

ARFIMA models encompass the traditional ARMA-ARIMA set-up but, in addition, they offer other interesting possibilities to model the persistence of shocks. The fundamental long-term properties of ARFIMA processes are governed by the fractional order of integration, d, and can be described in terms of interval regions for this parameter. For values of $d \in (0, 0.5)$, Q_t is said to be a *long-memory* process, characterized by slowly decaying (nonsummable) autocorrelations: $\rho(k) \sim ck^{2d-1}$ for large k, where $\rho(.)$ represents the autocorrelation function and c is a constant. In other words, long memory is implied by a hyperbolic decay of correlations, as opposed to the case of $d = 0$ where correlations decay exponentially. If $d \in [0.5, 1)$, Q_t is non-stationary (it has unbounded variance) and yet mean reverts, in the sense that shocks eventually die out. Values of d greater than or equal to 1 imply a permanent behavior of shocks.

As is customary in the literature, we measure persistence by computing impulse response functions (IRFs) and half lives (HLs). The IRF measures the effect of a shock of size one at time t on future values of the variable of interest, while the HL is defined as the number of periods that a shock needs to vanish by 50 percent and can be easily calculated from the IRF as[5]

$$IRF(HL) = 0.5. \tag{8}$$

For ARFIMA models, the IRF(h) can be computed as the h-th coefficient of the polynomial $A(L) = (1-L)^{-d}\,\Phi(L)^{-1}\,\Theta(L)$. These coefficients are given by

$$IRF(h) = \sum_{i=0}^{h} \pi_i(-d)\,J(h-i),$$

where the $\pi_i(-d)$'s come from the binomial expansion of $(1-L)^{-d}$ in powers of L (see (6)), and $J(.)$ is the standard ARMA(p,q) impulse response, given by $J(i) = \sum_{j=0}^{q} \theta_j f_{i+1-j}$, with $\theta_0 = 1$, $f_h = 0$ for $h \leq 0$, $f_1 = 1$ and $f_h = -(\phi_1 f_{h-1} + ... + \phi_p f_{h-p})$, for $h \geq 2$.

As first noticed by Hauser et al. (1999), despite their flexibility, ARFIMA models present a drawback when employed to evaluate persistence. The long-term impact of a shock happening at time t, as measured by the IRF(h) as $h \to \infty$, critically depends on the value of d. If $d < 1$, then $IRF(\infty) = 0$; if $d = 1$, $IRF(\infty)$ equals a constant (given by $\Theta(1)/\Phi(1)$) and, finally, whenever $d > 1$, IRF$(\infty) = \infty$.

Based on the behavior of the IRF at ∞, Hauser et al. (1999) argue that the estimated long-term effect obtained from an ARFIMA specification will necessarily be 0 or ∞ since it will be extremely unlikely to obtain an integer value of d even when the true d is, in fact, an integer. Although, in this paper, the focus is not on the long-run effect of shocks, it is

[5]Since the HL is nonmonotonic for general ARFIMA processes, in practice, it is calculated as the largest value that verifies that $IRF(HL-1) \geq 0.5$ and $IRF(HL+1) < 0.5$.

reasonable to think that this long-run behavior could impose constraints on other values of the IRF.

The Bayesian approach adopted in this paper, similar to that employed in Koop et al. (1997), will allow us to avoid this criticism by attaching a prior probability mass to ARMA, ARIMA and ARFIMA specifications (that is, to integer and fractional values of d). We do not impose a priori a fractional integration order for RERs as, in practice, the classical estimation approach does. In this way, it will be possible to obtain non-degenerate distributions for any value of the IRF and, in particular, of $IRF(\infty)$.

3 Modeling Aggregate real exchange rate data

This section presents the data and explores the evidence in favor of the holding of the PPP taking model uncertainty into consideration. This is possible by adopting a Bayesian approach. For completeness, this section opens with a standard Classical analysis of RER data. A comparison of the results obtained using Classical and Bayesian methods will highlight the hazard of choosing just one model, as the Classical approach does.

3.1 The data

We employ the database elaborated by Taylor (2002) consisting of a sample of 20 countries over a period running from 1850 to 1996 (although some series start later). To extend the time span (to 2004), we have used data from the IMF's *International Financial Statistics* . The countries included in the study are Argentina (ARG), Australia (AUS), Belgium (BEL), Brazil (BRA), Canada (CAN), Chile (CHL), Denmark (DNK), Finland (FI), France (FR), Germany (GE), Italy (IT), Japan (JPN), Mexico (MEX), Netherlands (NLD), Norway (NOR), Portugal (PRT), Spain (SP), Sweden (SWE), Switzerland (SWI) and Great

Britain (GBR).[6] The data is annual and RERs have been constructed as

$$Q_t = P_t - P_t^{US} - e_t$$

where P_t and P_t^{US} are the logs of the price index for the domestic country and the U.S.

3.2 Classical analysis

We begin our study by carrying out a standard classical analysis. To save space, we have collected the results in the Appendix.

To obtain model estimates under the classical approach, one first needs to choose a '*best*' model (typically selected by testing for the order of integration and using information criteria) and then, obtain estimates and standard errors as if the model was known a priori. An important drawback of this approach is that the uncertainty related to the model selection process is not adequately reflected in the standard errors which, as a consequence, are underestimated.

Standard unit root and stationary tests, e.g., the MZt-GLS (Ng and Perron, 2001) and the KPSS (Kwiatkowski et al., 1992) are applied to the data described above. These tests set the I(1) and I(0) models, respectively, as null hypothesis. Table A1 in the Appendix presents the results, which vary greatly across countries. The existence of a unit root can be rejected in most cases (14 out of 20). Nonetheless, the hypothesis of I(0) is also rejected for 11 countries in the sample and, surprisingly, both the I(1) and the I(0) hypotheses are rejected for 6 countries in our dataset.

These ambiguous results have often been interpreted as an indicator of a behavior midway between the I(0) and the I(1) models. Hence, what follows, we explicitly allow

[6] Following Taylor (2002), missing values for specific periods such as the World Wars and some hyper-inflation episodes are interpolated into the series using the Tramo-Seats program (Gómez and Maravall, 1996).

for fractional orders of integration. Tests of integer versus fractional integration orders are considered. To test the null hypothesis of $d = 1$ versus $d < 1$, the Fractional Dickey-Fuller (FDF) test are applied (Dolado et al., 2002, 2006). This test generalizes the traditional Dickey-Fuller approach to test for $I(1)$ against $I(0)$ to the more general framework of $I(1)$ versus FI(d), with $d < 1$.[7] The results are reported in Table A2 in the Appendix. The conclusion of this table is clear: the unit root model is clearly rejected against fractionally integrated alternatives in all countries, with the exception of Japan, in a regression where the only deterministic component is an intercept. If a trend is also introduced into the regression, the $I(1)$ hypothesis can also be rejected for Japan. Analogously, tests of the hypotheses FI(d) vs. $I(0)$ are also computed following a similar approach (Dolado et al., 2006) and, with the exception of Finland, the null hypothesis of FI(d) could not be rejected in any case.[8]

Results in Table A2 provide strong support in favor of fractional integration in RER data. However, several studies have pointed out that some types of nonlinearities can induce a strong persistence in the autocorrelation function and hence generate "spurious" FI (Diebold and Inoue, 2001, Granger and Hyung, 2004, Perron and Qu, 2006, etc.). Since our series cover more than a century we can not discard the possibility of structural breaks. To address this issue, we have applied two tests designed to distinguish between "true" and "spurious" FI (Shimotsu, 2006), that have power against different forms of nonlinearities.[9]

[7] The test is based on the t-ratio associated with the coefficient of $(1-L)^d y_{t-1}$ in a regression of $(1-L) y_t$ on $(1-L)^d y_{t-1}$ and, possibly, some lags of $(1-L) y_t$ to account for the short-run autocorrelation of the process and/or some deterministic components if the series displays a trending behavior or initial conditions different from zero. To compute the tests, an estimated value of d under H_1 is required and the values of the EML estimator, reported in the fourth column of Table A4, have been used.

[8] The results of these tests have been omitted for the sake of brevity, since similar conclusions can also be drawn from tests based on the confidence intervals of the memory parameter, d, reported in Table A4 below.

[9] These tests exploit two time-domain properties of FI(d) processes. First, if a time series follows a FI(d) process, then each subsample is also FI with the same value of d. Thus, the first test splits the sample into b subsamples and compares the estimated values of d in each subsample with the estimate for the full sample. The second test is based on the fact that the dth difference of a FI(d) process follows an I(0) process.

Results are presented in Table A3 in the Appendix. This table shows that, in general, there is robust evidence of FI behavior in RER data. The null hypothesis of FI is rejected only for two countries (Brazil and Mexico) out of the 20 countries considered in this study and only by one of the three tests considered in this analysis.

Next, FI models have been fitted to the aggregate RERs. Several approaches have been considered: the semi-parametric feasible exact local Whittle estimator (FELW, Shimotsu, 2010)[10] and the parametric exact maximum likelihood (EML) and non-linear least squares methods (NLS) proposed by Sowell (1992) and Beran (1994), respectively.

The 'memory' parameter, d, contains valuable information about the nature of PPP deviations. PPP holds if deviations from parity have a transitory character, that is, if Q_t is a mean-reverting process. For this to happen, it is not necessary that $d = 0$, as is frequently imposed in the literature. If $d < 1$, shocks to Q_t have a transitory character and thus, PPP holds in the long-run. However, values of d close to 1 will indicate that the convergence to parity takes place very slowly.

Estimated values of d are reported in Table A4. Although the results vary slightly across the different methods and countries, fractional values of d, far from both 0 and 1, are found in general. Most countries exhibit values of d in the region of $0.5 \leq d < 1$. In these cases, Q_t is mean-reverting but non-stationary and, thus, the rate of convergence to equilibrium is very slow. This is in agreement with previous findings about the nature of shocks to RER, which are usually characterized as transitory but very persistent.

Summarizing, the classical approach provides overwhelming evidence in favor of fractional integration for aggregate RERs and the long-run holding of the PPP.

Then, KPSS and Phillips-Perron tests are applied to the fractionally-differenced data and its partial sum, respectively. Shimotsu (2006) shows that these simple tests have power against many types of nonlinear models (such as models with structural breaks, Stopbreak models, Markov switching models, etc.) See his paper for additional details.

[10]The FELW is a version of the local Whittle estimator (Robinson, 1995) and is consistent even if the DGP is non-stationary and contains deterministic components.

3.3 Bayesian analysis

Classical estimators of ARFIMA models are extremely sensitive to the parametric model employed and to the choice of bandwidth when semi-parametric estimators are considered. However, this uncertainty is not adequately reflected in the estimates' standard errors since, once a model (or the bandwidth) is selected, it is considered as given.

To overcome this problem, we have re-estimated the series adopting a Bayesian approach. The method has several advantages over the classical estimation in our context. First, it allows us to average across models rather than relying on just one, which will translate into more accurate measures of the uncertainty involved in persistence estimates. Second, it is possible to attach some prior probability mass to integer values of d ($\{d = 0, 1\}$), which implies that ARMA, ARIMA and ARFIMA specifications can be put on an equal footing. As mentioned above, this is particularly important when ARFIMA models are employed for measuring persistence. And third, the method provides exact finite-sample distributions, and not just standard errors, for any feature of interest (IRFs, HLs, values of d, etc). This is important since finite-sample distributions can be very different from asymptotic ones. For instance, finite-sample densities of IRFs have been shown to be multimodal, highly skewed and fat-tailed and their moments may even not exist (Koop et al., 1994). In addition, this method will allow us to compute interesting probabilities, such as the probability of the holding of the PPP, $P(d < 1)$, or the probability of the so-called Rogoff's interval (HLs in the interval of 3 to 5 years), among others.

To obtain the Bayesian estimates, the method proposed by Koop et al. (1997) is closely followed .[11] Twenty-seven models are estimated, corresponding to all possible combinations of ARMA(p,q), ARFIMA(p,d,q) and ARIMA(p,1,q) models with p, $q \leq 2$. These models are derived by putting constraints on the unrestricted ARFIMA$(2, d, 2)$ specification. This

[11] See Koop et al. (1997) for details of the estimation procedure. Bayesian estimates have been obtained using a Fortran code kindly provided by the authors.

implies that functions of the parameters, such as values of d, IRFs or HLs are not model-specific quantities and, thus, it is possible to average them over models (see Min and Zellner, 1993).[12] A flat prior probability is attached to each model so that the method puts an equal $1/3$ of the prior probability mass on fractional values of d, $d = 0$, $d = 1$. In the former case, a uniform density for d in the interval $(0, 1.5)$ is assumed.

Table I reports the total posterior probabilities of the ARMA, ARFIMA and ARIMA specifications. The ARFIMA formulation is the one with the highest posterior probability (50%, on average). The ARMA and ARIMA specifications also obtain considerable posterior probabilities: 0.27 and 0.23, respectively. This implies that, although there is evidence in favour of fractional orders of integration, it is not overwhelming. In addition, the posterior model probability is quite scattered across all the models considered, which shows the hazard of selecting just one.[13] It follows that the uncertainty involved in the model selection process seems to be considerably larger than what the classical analysis seems to suggest.

(Table I about here)

Table II presents Bayesian estimates of the order of integration corresponding to the models analyzed in the previous table. It provides the median and the 2.5 and 97.5 percentiles, computed from four different distributions of $\hat{d}$, denoted as *best-all, best-ARFIMA, weighted-all* and *weighted-ARFIMA*. '*Best-all*' and '*best-ARFIMA*' correspond to the densities of d associated with the models that obtain the highest posterior probability of all the models and of all the ARFIMA specifications, respectively. '*Weighted-all*' and

[12]To compare models, proper, normalized prior densities for free parameters other than location and scale are required. This is fulfilled by proper uniform priors for the free ARFIMA coefficients which, as advocated by, e.g., Poirier (1985), provide an overall coherent prior structure, see Koop et al. (1997) for additional details.

[13]The corresponding figures are not reported for the sake of brevity but are available upon request.

'Weighted-ARFIMA' refer to the weighted average of the densities of $\hat{d}$ corresponding to all the models and all the ARFIMAS, respectively, where the weights are the posterior probabilities assigned to each model.[14]

As shown in Table II, the median values of $\hat{d}$ obtained in the "*'weighted'* distributions are, in general, greater than those corresponding to the '*best'* models. But, more importantly, uncertainty, as measured by the length of the 2.5 and 97.5 percentile interval, is considerably larger when weighted distributions are considered. The average length of this interval increases by 63 and 188% when comparing the *best-ARFIMA* and the *best-all* estimations to the *weighted-ARFIMA* and *weighted-all* ones, respectively. Thus, by averaging the models, larger and more spread out values of d are obtained.

(Table II about here)

As mentioned above, a further advantage of the Bayesian approach adopted in this section is that it allows us to assess uncertainty by computing interesting probabilities about the holding of PPP and the nature of the RERs. Table III provides the probabilities that d belongs to certain intervals of interest. Calculations have been carried out using the '*weighted-all'* distribution of $\hat{d}$ in order to obtain more accurate estimates. This table presents the probability that PPP holds ($d < 1$) and the probabilities that the RERs are $I(0)$, stationary ($d < 0.5$) or contain an exact unit root ($d = 1$) . For comparison, the Appendix contains similar figures computed from the '*best-all'* and the '*best-ARFIMA'* densities.

The results indicate that the uncertainty involved in the estimation of d is large, as shown by the fact that its distribution is quite spread out. In spite of this, we should

[14] Posterior properties of the parameters are calculated using Monte Carlo integration with 25,000 replications and importance sampling (Geweke, 1989). Both the parameter priors and the importance function are taken to be proper uniform densities of the parameter space (see Koop et al. (1997) for details)

highlight that the posterior probability of the holding of PPP is, on average, quite large, reaching 0.72. However, there is considerable cross-country heterogeneity. While, for some countries, such as Argentina, Belgium, Sweden and Finland, the $P(d < 1)$ is larger than 90%, for others, such as Germany, Japan, Canada, Brazil and the Netherlands, it is only around 50%.

If the above-described '*best*' models are employed to compute this probability, the uncertainty about the holding of PPP decreases significantly. According to the *best-ARFIMA* model, the $P(d < 1)$ is, on average, 0.98. It follows that, if no prior probability were associated with integer values of d and model selection uncertainty were not recognized, evidence in favour of the holding of PPP would be overwhelming, a result that closely resembles that found in the previous section when classical techniques were employed.[15] However, the fact that the difference between these two probabilities (0.98 and 0.72) is substantial underscores the importance of considering model selection uncertainty if the goal is to produce more realistic estimates about the holding of PPP.

Finally, the probability that the RERs are non-stationary appears to be large (51% according to the '*weighted-all*' posterior density). This implies that, if PPP holds, convergence to parity is likely to take place very slowly.

(Table III about here)

4 IRFs and Half-lives

Section 3 shows that, in spite of the high uncertainty involved in RER estimation, PPP is likely to hold in the long run for most of the countries considered in this study, in line with recent literature. This section investigates how the convergence to equilibrium takes

[15] According to the *best-all* model, the $P(d < 1)$=0.81. See Table A5 in the Appendix.

place. To do so, IRFs and HLs have been computed.[16]

Table IV reports the median and the 2.5 and 97.5 percentiles of the posterior distribution of the HL corresponding to the *best-ARFIMA* and the *weighted-all* posterior distributions. For a more detailed picture of the whole distribution of the HL, Tables V and VI provide posterior probabilities of the HL lying in some regions of interest obtained from the *weighted-all* and the *best-ARFIMA* posterior distributions, respectively. Finally, Figures 1 and 2 display the *weighted-all* and the *best-ARFIMA* posterior cumulative distributions of the HLs.[17] To simplify the calculations, values of the HL larger than 10 years are grouped into one category.

The median values of the HL are very heterogeneous across countries but, in general, are quite large. According to the '*weighted-all*' posterior distributions, they are larger than 10 years for 10 countries in the sample. Moreover, 17 countries have a 97.5 percentile that is also above this quantity. This confirms the very persistent character of deviations from PPP. Table V shows that, on average, the probability that the HL is smaller than 10 years is 55% according to the '*weighted-all*' distribution. However, there exists considerable within-country heterogeneity: for some countries (ARG, BEL, FIN, FRA, ITA, SWE) this probability is close to 1 while for others (like GE, JPN, NLD) it is below 20%. Not surprisingly, the probability that the HL lies in the so-called Rogoff's interval (3 to 5 years) is, in general, small (21% on average), being only larger than 50% for two countries (ITA and SWE). The results obtained from the *best-ARFIMA* posterior density are similar, although the uncertainty tends to be somewhat smaller. Table V shows that only 7 countries display median values of the HL that are larger than 10 years while, from Table VI, it seems that the $P\,(HL < 10)$ years) equals 60%.

[16] A detailed revision of persistence measures in a fractional integration context can be found in Gadea and Mayoral (2006).

[17] The method provides the distribution function of HLs for some integer horizons, h=1,2,...,10. the full function is interpolated by a cuadratic spline function.

Finally, Figures 1A and 1B underscore the high uncertainty associated with the estimation of the HL. For most countries, the distributions appear to be quite scattered over the different values, even more so when the '*weighted-all*' distribution is employed.

The results described above differ considerably from those reported in the original work of Taylor (2002), who obtains a mean HL of around 4 years. Our findings are closer to those of Murray and Papell (2002, 2005) and Lopez et al. (2005, 2013) who, using a different methodology, also find HLs above the 3-5 Rogoff consensus. Using Taylor's (2002) database and median-unbiased estimators, Lopez et al. (2013) find a median HL of 11.34 and 7.55 in the DF-GLS and ADF regressions, respectively.

In sum, our results suggest that, given the high uncertainty involved in the estimation of aggregate RERs, it is difficult to obtain robust estimators of the HLs. However, the data seems to be informative about several issues. PPP is likely to hold in the long run for most of the countries considered in this paper, as shown in the previous section, although the rate of reversion to parity seems to be very slow, with HLs that appear to be considerably larger than 3-5 years.

(Table IV about here)

(Table V about here)

(Table VI about here)

5 Conclusions

In recent years, there has been a rebirth of interest in the sources and measurement of PPP deviations. However, in spite of the large quantity of literature on the subject, the debate is still open. Since the so-called Rogoff's puzzle was formulated, recent research has questioned the validity of Rogoff's remarkable consensus of 3-5 year half-lives of deviations from PPP. Some critics stress the high uncertainty of persistence estimates and the heterogeneity present in the dynamics of sectoral RERs.

This paper uses a Bayesian approach to assess the uncertainty of estimates involved in the holding of PPP and the speed of reversion to parity. The novelty of the paper is the combination of a very flexible modelization of RERs that takes into account the existence of heterogeneous dynamics in a parsimonious way, and the Bayesian methodology that allows us to obtain a new perspective of the uncertainty involved in the estimation of the quantities of interest. This framework has been applied to a wide database that includes the RERs of 20 countries over more than a century.

Using a wide range of classical estimators, empirical evidence of fractional integration in the RERs with orders of integration that are, in general, larger than 0 but smaller than 1 is found. Nevertheless, using a Bayesian approach shows that the model uncertainty is very high. The evidence in favor of fractional integration is not overwhelming and ARMA and ARIMA models, corresponding to integration orders of 0 and 1, respectively, obtain non-trivial posterior probabilities. Furthermore, the distribution of posterior probabilities across the model considered highlights the role of chance in the selection model.

There is also a lot of uncertainty both about the value of the order of integration and, especially, about the size of the HL. The speed of reversion seems to be slower than that found in the original work of Taylor. The probability that the HL lies in the so-called Rogoff's puzzle interval (3-5 years) is quite small (around 21%). Our results are very much in line with those presented in Murray and Papell (2002) and Lopez et al. (2005, 2013), obtained with bias-corrected estimates, Rossi (2005), who computes confidence intervals using local-to-unity asymptotic theory and Kilian and Zha (2001), who use a Bayesian approach in AR(p) models.

In addition, we show that their measures of uncertainty can suffer from an important bias if model uncertainty is not taken into account. In spite of this high uncertainty, our results find robust support to re-establish the PPP hypothesis as a valid rule for the very long run.

Finally, one limitation of our study is that only linear models are considered in the Bayesian analysis. Although we provide (frequentists) tests displaying support for FI versus models with nonlinearities, a more comprehensive analysis would involve considering both FI and nonlinear models under the Bayesian analysis, letting the data choose the most likely model in each case. Since the technical difficulties associated to this approach are considerable, we leave this issue for future research.

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Tables

TABLE I
Posterior Model Probabilities

	ARMA	ARFIMA	ARIMA
ARG	0.58	0.38	0.04
AUS	0.17	0.64	0.18
BEL	0.58	0.38	0.03
BRA	0.19	0.45	0.36
CAN	0.12	0.54	0.34
CHL	0.16	0.59	0.25
DNK	0.16	0.61	0.23
FIN	0.68	0.28	0.04
FRA	0.16	0.61	0.23
GE	0.17	0.36	0.47
ITA	0.40	0.54	0.05
JPN	0.11	0.46	0.44
MEX	0.34	0.53	0.13
NLD	0.12	0.50	0.38
NOR	0.44	0.39	0.17
PRT	0.07	0.58	0.35
SP	0.06	0.59	0.35
SWE	0.53	0.40	0.08
SWI	0.17	0.52	0.31
GRB	0.15	0.66	0.19
Mean	**0.27**	**0.50**	**0.23**

Notes. This table displays the posterior probabilities associated with the ARMA, (pure) ARFIMA and ARIMA specifications. 25,000 replications of the 27 possible combinations of ARFIMA(p,d,q), ARIMA(p,1,q) and ARMA(p,q) with p,q $\leq$ 2 and $d \in$ (0,1.5) have been employed to obtain the Bayesian estimates. Posterior model probabilities have been obtained as described in Koop et al. (1997).

TABLE II

Bayesian Estimation of d

	best-ARFIMA	best-all	weighted-ARFIMA	weighted-all
ARG	0.25 (0.03,0.68)	0 (0)	0.38 (0.03,1.05)	0 (0,0.99)
AUS	0.51 (0.13,0.90)	0.51 (0.13,0.90)	0.63 (0.10,1.25)	0.63 (0,1.16)
BEL	0.32 (0.09,0.63)	0 (0)	0.37 (0.02,1.1)	0 (0,0.99)
BRA	0.53 (0.04,1.03)	0.53 (0.04,1.03)	0.75 (0.08,1.25)	0.89 (0,1.14)
CAN	0.50 (0.10,1.03)	0.50 (0.10,1.03)	0.71 (0.10,1.26)	0.88 (0,1.17)
CHL	0.74 (0.56,0.95)	0.74 (0.56,0.95)	0.64 (0.10,1.30)	0.69 (0,1.19)
DNK	0.51 (0.13,0.92)	0.51 (0.13,0.92)	0.65 (0.10,1.29)	0.69 (0,1.20)
FIN	0.26 (0.03,0.66)	0 (0)	0.33 (0.03,1.07)	0 (0,0.83)
FRA	0.39 (0.05,0.82)	0.39 (0.05,0.82)	0.55 (0.11,1.32)	0.59 (0,1.23)
GE	0.51 (0.06,1.10)	1 (1)	0.81 (0.08,1.39)	0.98 (0,1.32)
ITA	0.40 (0.08,0.83)	0 (0)	0.48 (0.03,1.16)	0.21 (0,1.15)
JPN	0.81 (0.61,1.05)	1 (1)	0.75 (0.14,1.34)	0.98 (0,1.21)
MEX	0.55 (0.39,0.84)	0 (0)	0.57 (0.05,1.30)	0.45 (0,1.12)
NLD	0.74 (0.28,0.87)	0.74 (0.28,0.87)	0.71 (0.09,1.30)	0.93 (0,1.24)
NOR	0.64 (0.32,1.02)	0 (0)	0.65 (0.08,1.35)	0.32 (0,1.27)
PRT	0.58 (0.16,0.98)	0.58 (0.16,0.98)	0.66 (0.16,1.23)	0.85 (0,1.15)
SP	0.58 (0.14,0.96)	1 (1)	0.72 (0.23,1.32)	0.90 (0,1.01)
SWE	0.29 (0.03,0.72)	0 (0)	0.51 (0.04,1.24)	0.10 (0,1.08)
SWI	0.66 (0.29,1.06)	0.66 (0.29,1.06)	0.67 (0.10,1.33)	0.77 (0,1.27)
GRB	0.50 (0.10,0.88)	0.50 (0.10,0.88)	0.66 (0.13,1.34)	0.67 (0,1.27)
Mean	**0.51**	**0.43**	**0.61**	**0.58**

Notes. This table presents the median values and 2.5 and 97.5 percentiles obtained from four posterior distributions of d : best-ARFIMA, best-all, weighted-ARFIMA and weighted-all.

TABLE III

POSTERIOR PROBABILITIES OF d

	P($d = 0$)	P($d < 0.5$)	P($0.5 \leq d < 1$)	P($d < 1$)	P($d = 1$)	P($d > 1$)
ARG	0.58	0.84	0.09	0.93	0.04	0.03
AUS	0.17	0.43	0.36	0.79	0.18	0.03
BEL	0.58	0.89	0.07	0.96	0.03	0.00
BRA	0.19	0.30	0.26	0.55	0.36	0.09
CAN	0.12	0.29	0.26	0.54	0.34	0.11
CHL	0.16	0.26	0.45	0.71	0.25	0.04
DNK	0.16	0.38	0.33	0.71	0.23	0.06
FIN	0.68	0.92	0.01	0.93	0.04	0.03
FRA	0.16	0.48	0.25	0.73	0.23	0.04
GE	0.17	0.27	0.17	0.44	0.47	0.09
ITA	0.40	0.74	0.20	0.93	0.05	0.02
JPN	0.11	0.19	0.31	0.51	0.44	0.05
MEX	0.34	0.57	0.27	0.85	0.13	0.02
NLD	0.12	0.27	0.31	0.58	0.38	0.04
NOR	0.44	0.61	0.20	0.81	0.17	0.02
PRT	0.07	0.23	0.37	0.61	0.35	0.04
SP	0.06	0.18	0.41	0.59	0.35	0.05
SWE	0.53	0.76	0.15	0.91	0.08	0.01
SWI	0.17	0.35	0.31	0.65	0.31	0.04
GRB	0.15	0.37	0.39	0.76	0.19	0.05
Mean	**0.27**	**0.47**	**0.26**	**0.72**	**0.23**	**0.04**

Notes. This table provides the probabilities associated with d belonging to one of several intervals of interest. The weighted-all posterior probability has been employed to compute the probabilities.

TABLE IV

Median Half-lives

	best _ARFIMA	weighted _all
ARG	1.98 (1.03,5.15)	1.80 (3.78,8.93)
AUS	8.25 (3.60,>10)	8.70 (3.15,>10)
BEL	2.78 (2.00,9.45)	2.73 (1.95,4.63)
BRA	8.80 (3.48,>10)	> 10 (3.48,>10)
CAN	> 10 (4.15,>10)	> 10 (4.13,>10)
CHL	6.08 (1.13,>10)	6.58 (1.38,>10)
DNK	8.63 (3.63,>10)	> 10 (3.10,>10)
FIN	2.28 (1.53,3.08)	2.03 (1.08,3.10)
FRA	4.43 (2.35,>10)	4.00 (2.15,>10)
GE	> 10 (5.75,,>10)	> 10 (4.63,>10)
ITA	6.70 (3.08,>10)	4.45 (2.33>10)
JPN	> 10 (6.55,>10)	> 10 (4.83,>10)
MEX	3.10 (1.20,>10)	3.38 (1.45,>10)
NLD	> 10 (6.85,>10)	> 10 (4.95,>10)
NOR	> 10 (7.73,>10)	> 10 (4.05,>10)
PRT	> 10 (4.10,>10)	> 10 (3.38,>10)
SP	> 10 (3.53,>10)	> 10 (3.90,>10)
SWE	4.48 (2.05,>10)	3.98 (2.28,>10)
SWI	8.70 (3.78,>10)	> 10 (3.53,>10)
GRB	6.25 (2.95,>10)	7.63 (2.70,>10)

Notes. This table presents the median values and 2.5 and 97.5 percentiles obtained from the best-ARFIMA and weighted-all posterior distributions.

TABLE V

Posterior Probabilities of half-lives

	$HL \leq 2$	$HL \leq 3$	$HL \leq 5$	$3 \leq HL \leq 5$	$HL \leq 10$	$HL > 10$
ARG	0.57	0.81	0.95	0.15	0.98	0.02
AUS	0.00	0.02	0.23	0.19	0.54	0.46
BEL	0.04	0.57	0.89	0.43	0.96	0.04
BRA	0.00	0.01	0.12	0.11	0.31	0.69
CAN	0.00	0.00	0.06	0.06	0.28	0.72
CHL	0.10	0.26	0.45	0.19	0.62	0.38
DNK	0.00	0.02	0.21	0.19	0.49	0.51
FIN	0.48	0.94	0.98	0.17	0.99	0.07
FRA	0.00	0.21	0.67	0.47	0.82	0.18
GE	0.00	0.00	0.04	0.06	0.18	0.82
ITA	0.00	0.13	0.58	0.51	0.82	0.18
JPN	0.00	0.00	0.03	0.03	0.19	0.81
MEX	0.13	0.41	0.66	0.30	0.76	0.24
NLD	0.00	0.00	0.03	0.05	0.20	0.80
NOR	0.00	0.00	0.10	0.13	0.45	0.55
PRT	0.00	0.01	0.13	0.13	0.33	0.67
SP	0.00	0.00	0.08	0.08	0.25	0.75
SWE	0.00	0.17	0.68	0.55	0.85	0.15
SWI	0.00	0.00	0.14	0.15	0.38	0.62
GRB	0.00	0.05	0.30	0.25	0.58	0.42
Mean	**0.07**	**0.18**	**0.37**	**0.21**	**0.55**	**0.45**

Notes. This table provides the probability that the HL belongs to one of several intervals of interest. The weighted-all posterior probability has been employed to compute these probabilities.

TABLE VI

Posterior Probabilities of half-lives

	$HL \leq 2$	$HL \leq 3$	$HL \leq 5$	$3 \leq HL \leq 5$	$HL \leq 10$	$HL > 10$
ARG	0.50	0.87	0.97	0.10	0.99	0.01
AUS	0.00	0.00	0.15	0.15	0.61	0.39
BEL	0.01	0.61	0.89	0.28	0.98	0.02
BRA	0.00	0.01	0.16	0.15	0.55	0.45
CAN	0.00	0.00	0.08	0.07	0.49	0.51
CHL	0.13	0.24	0.44	0.20	0.64	0.36
DNK	0.00	0.00	0.14	0.14	0.58	0.42
FIN	0.29	0.95	0.98	0.04	0.99	0.01
FRA	0.01	0.11	0.60	0.49	0.88	0.12
GE	0.00	0.00	0.01	0.01	0.23	0.77
ITA	0.00	0.02	0.27	0.25	0.74	0.26
JPN	0.00	0.00	0.01	0.01	0.06	0.94
MEX	0.20	0.47	0.73	0.25	0.89	0.11
NLD	0.00	0.00	0.00	0.00	0.11	0.89
NOR	0.00	0.00	0.00	0.00	0.08	0.92
PRT	0.00	0.00	0.07	0.07	0.43	0.57
SP	0.00	0.00	0.16	0.15	0.46	0.54
SWE	0.00	0.09	0.61	0.51	0.88	0.12
SWI	0.00	0.00	0.16	0.15	0.57	0.43
GRB	0.00	0.03	0.32	0.29	0.74	0.26
Mean	**0.06**	**0.17**	**0.34**	**0.17**	**0.60**	**0.40**

Notes. This table provides the probability that the HL belongs to one of several intervals of interest.The weighted-all posterior probability has been employed to compute these probabilities.

Figures

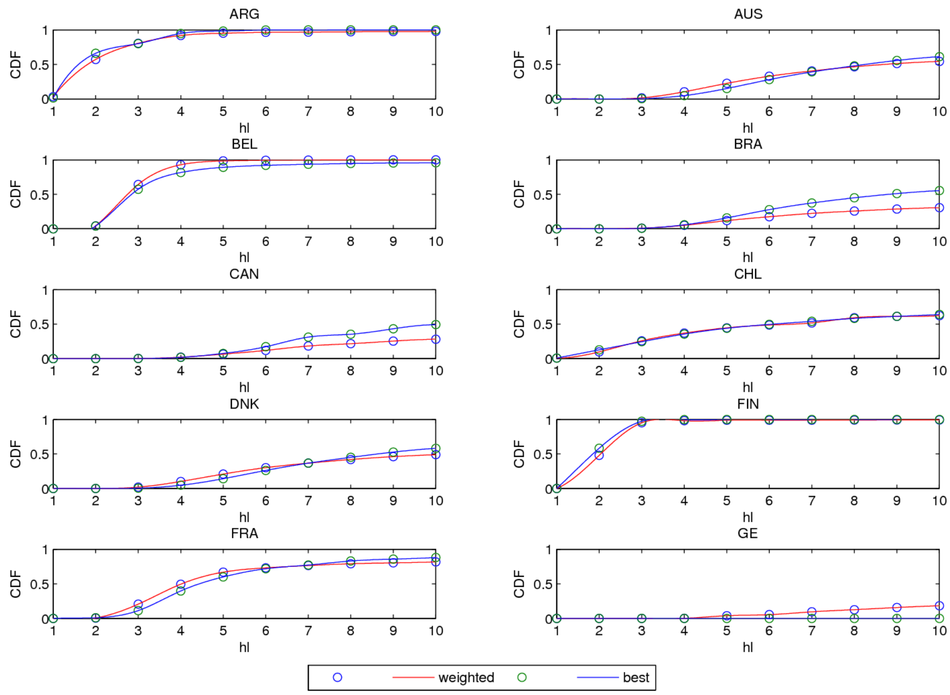

FIGURE 1A. HL posterior cumulative distribution function

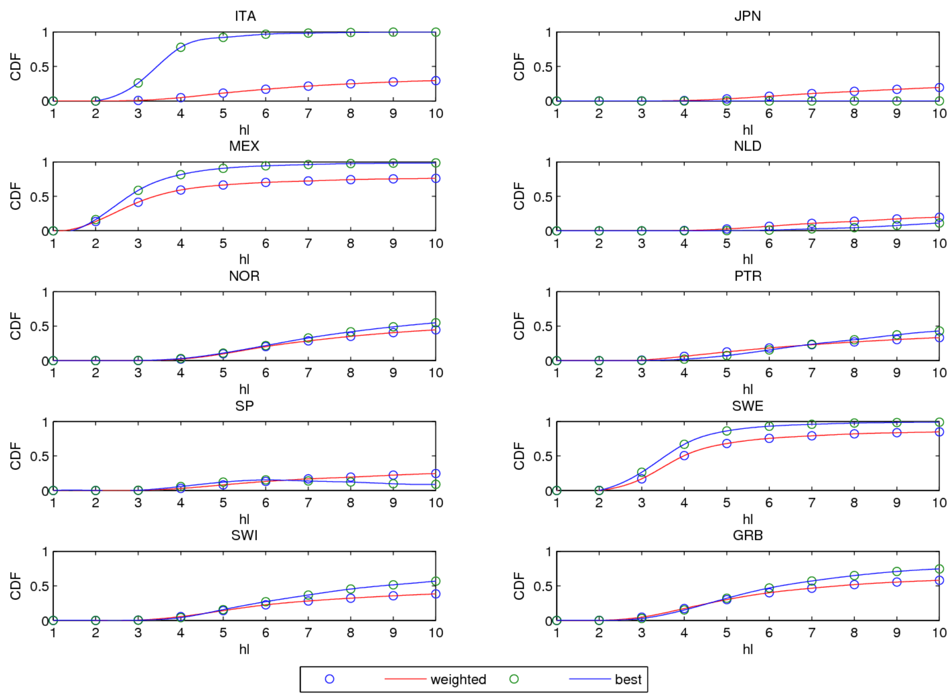

FIGURE 1B. HL posterior cumulative distribution function

Appendix

TABLE A1

Unit root and stationarity tests

	MZ_t-GLS	KPSS
ARG	-4.09**	0.11
AUS	-2.43**	0.81**
BEL	-3.37**	0.74**
BRA	-2.53**	0.16
CAN	-1.77	0.78**
CHL	-1.02	1.06**
DNK	-2.00	0.68**
FIN	-5.70**	0.18
FRA	-2.33*	0.89**
GE	-2.67**	0.39
ITA	-3.99**	0.07
JPN	0.17	1.14**
MEX	-2.50*	0.81*
NLD	-2.58**	0.43
NOR	-2.37*	0.46*
PRT	-1.85	0.40
SP	-2.87**	0.31
SWE	-3.21**	0.59*
SWI	-0.68	0.94**
GBR	-2.58**	0.38

Notes. This table presents the results of the MZt-GLS and the KPSS tests. (**) and (*) denote rejection at the 1% and 5% level, respectively. An intercept was included to compute the statistics. The SBIC and Bartlett's window (with the bandwidth chosen according to Newey and West, 1994) have been used in the computation of the MZt-GLS and KPSS tests, respectively.

TABLE A2

FDF Test (I(1) versus FI(d)).

	$H_0 : d = 1$; $H_1 : d < 1$	
	with intercept	with intercept and trend
ARG	-4.47**	-4.53**
AUS	-2.99**	-3.41**
BEL	-4.10**	-4.46**
BRA	-2.57**	-2.04*
CAN	-2.99**	-3.31**
CHL	-2.26*	-2.96**
DNK	-2.74**	-3.45**
FIN	-6.20**	-6.24**
FRA	-4.66**	-4.75**
GE	-2.43*	-2.47*
ITA	-3.87**	-4.08**
JPN	-0.97	-3.01**
MEX	-3.54**	-3.56**
NLD	-2.55*	-2.83**
NOR	-4.22**	-4.56**
PRT	-1.68*	-3.61**
SP	-3.50**	-3.61**
SWE	-3.99**	-4.12**
SWI	-2.95*	-3.60*
GRB	-2.29*	-3.31*

Notes. This table provides the results of the FDF tests. (**) and (*) denote rejection at the 1% and the 5% level, respectively. Critical values: N(0,1). The FDF invariant regression $\Delta y_t = \alpha_1 \tau_{t-1}(d) + \phi \Delta^d y_{t-1} + \sum_{j=1}^{k} \Delta y_{t-j} + a_t$ is run and the number of lags of Δy_t is chosen according to the AIC. The term $\tau_t(\delta)$ is defined as $\tau_t(\delta) = \sum_{i=0}^{t-1} \pi_i(\delta)$, where the coefficients $\pi_i(\delta)$ come from the power expansion of $(1-L)^{\delta}$.

TABLE A3

FI VS NONLINEAR MODELS

	$\widehat{d}$	$\overline{d}$		W_c		Z_t	$\widehat{\eta_\mu}$
		$b=2$	$b=4$	$b=2$	$b=4$		
ARG	0.49	0.75	0.99	3.56	1.21	−2.59	0.05
AUS	0.81	0.82	1.10	0.21	0.94	−1.77	0.07
BEL	0.65	0.76	1.05	1.47	5.00	−1.54	0.07
BRA	0.86	0.98	1.35	4.81*	10.87*	−2.43	0.06
CAN	0.92	0.85	1.22	2.87	10.54	−2.00	0.06
CHL	0.83	0.90	1.05	2.91	3.89	−0.51	0.08
DNK	0.83	0.86	0.91	2.40	4.31	−1.46	0.11
FIN	0.35	0.56	0.84	3.65	5.60	−2.47	0.06
FRA	0.70	0.71	0.90	0.02	2.15	−1.55	0.07
GE	0.98	1.05	1.67	0.28	9.05	−2.51	0.04
ITA	0.79	0.89	0.95	1.94	3.40	−3.19	0.03
JPN	0.90	1.03	1.19	0.54	6.67	−0.35	0.08
MEX	0.68	0.70	0.94	5.83*	5.31*	−1.25	0.05
NLD	0.95	0.94	1.21	0.59	5.18	−2.19	0.10
NOR	0.90	0.98	1.22	1.17	4.71	−2.46	0.04
PRT	0.84	0.87	0.82	1.59	13.65	−2.08	0.12
SP	0.81	0.84	1.02	0.47	1.38	−2.00	0.13
SWE	0.73	0.83	1.01	0.40	0.49	−1.87	0.03
SWI	0.84	0.92	1.51	0.23	17.82	−0.68	0.06
GRB	0.72	0.76	0.86	0.28	0.62	−1.94	0.11

Notes. This table presents the results of applying the tests in Shimotsu (2006) to distinguish between FI and different nonlinear models. * indicates rejection of the null hypothesis of FI at the 5% level. $\hat{d}$ denotes the estimate of d in the full sample while $\overline{d}$ represents the estimates of d in each of the b subsamples. Estimates of d have been obtained using the exact local whittle estimator (Shimotsu, 2010). W_c is the test based on comparing the values of d estimated in the full sample and each of the subsamples. $\widehat{\eta_\mu}$ and Z_t denote the KPSS and the Phillips-Perron tests, respectively, applied to the fractionally differenced process and to its partial sum. The number of periodiogram ordinates is $m = t^{0.7}$ being t the sample size. Critical values for the W_c test are χ (1)=3.84, χ (3)=7.82 for $b = \{2, 4\}$. Critical values of Z_t and $\widehat{\eta}_\mu$ are computed using the Matlab code provided by Shimotsu (2006).

TABLE A4

Classical Estimation of FI(d) models

	FELW	EML	NLS
ARG	0.49 (0.088)	0.64 (0.096)	0.61 (0.097)
AUS	0.81 (0.088)	0.50 (0.187)	0.46 (0.181)
BEL	0.65 (0.088)	0.31 (0.090)	0.34 (0.103)
BRA	0.86 (0.088)	0.92 (0.090)	0.91 (0.090)
CAN	0.92 (0.088)	0.48 (0.137)	0.46 (0.213)
CHL	0.83 (0.088)	0.46 (0.261)	0.94 (0.151)
DNK	0.83 (0.088)	0.66 (0.108)	0.68 (0.102)
FIN	0.35 (0.088)	-0.30 (0.325)	-0.26 (0.312)
FRA	0.70 (0.088)	0.77 (0.283)	0.68 (0.171)
GE	0.98 (0.088)	0.17 (0.219)	0.55 (0.275)
ITA	0.79 (0.088)	0.43 (0.117)	0.39 (0.117)
JPN	0.90 (0.088)	0.47 (0.335)	0.90 (0.188)
MEX	0.68 (0.088)	0.50 (0.107)	0.53 (0.096)
NLD	0.95 (0.088)	0.34 (0.379)	0.42 (0.301)
NOR	0.90 (0.088)	0.53 (0.281)	0.32 (0.215)
PRT	0.84 (0.088)	0.73 (0.129)	0.56 (0.157)
SP	0.81 (0.088)	0.56 (0.106)	0.53 (0.106)
SWE	0.73 (0.088)	0.41 (0.135)	0.46 (0.122)
SWI	0.84 (0.088)	0.52 (0.125)	0.71 (0.158)
GRB	0.72 (0.088)	0.67 (0.100)	0.59 (0.106)

Notes. This table reports estimated values of d and standard errors obtained from three different estimation methods, FELW, EML and NLS. Computations for the EML and NLS estimators were carried out using the ARFIMA Package 1.04 for OX, (Doornik and Ooms, 2006) while the FELW was implemented using a MATLAB code provided by the author. Variables were estimated in first differences and 1 was added to the obtained value. Parametric models were selected using the AIC.

TABLE A5

POSTERIOR PROBABILITIES OF d

	P($d=0$)		P($d<0.5$)		P($0.5 \leq d < 1$)		P($d<1$)		P($d=1$)		P($d>1$)	
	best-all	best-ARFIMA	best-all	best-ARFIMA	best-all	best-ARFIMA	best-all	best-ARFIMA	best-all	best-ARFIMA	best-all	best-ARFIMA
ARG	1	0	1	0.88	0	0.18	1	1	0	0	0	0
AUS	0	0	0.38	0.48	0.54	0.45	0.92	0.99	0	0	0.08	0.01
BEL	1	0	1	0.87	0	0.13	1	1	0	0	0	0
BRA	0	0	0.29	0.47	0.64	0.49	0.93	0.96	0	0	0.07	0.04
CAN	0	0	0.26	0.49	0.65	0.48	0.91	0.97	0	0	0.08	0.03
CHL	0	0	0.33	0.00	0.61	0.99	0.94	0.99	0	0	0.06	0.01
DNK	0.00	0	0.34	0.47	0.58	0.52	0.92	0.99	0	0	0.08	0.01
FIN	1	0	1	0.89	0	0.11	1	1	0	0	0	0
FRA	0	0	0.42	0.71	0.49	0.28	0.91	0.99	0	0	0.09	0.01
GE	0	0	0	0.49	0	0.45	0	0.94	1	0	0	0.06
ITA	1	0	1	0.67	0	0.33	1	0.99	0	0	0	0.01
JPN	0	0	0	0.00	0	0.93	0	0.93	1	0	0	0.07
MEX	1	0	1	0.34	0	0.66	1	1	0	0	0	0
NLD	0.00	0	0.24	0.25	0.67	0.71	0.91	0.96	0	0	0.09	0.04
NOR	1	0	1	0.20	0	0.77	1	0.97	0	0	0	0.03
PRT	0.00	0	0.22	0.35	0.71	0.63	0.93	0.98	0	0	0.07	0.02
SP	0	0	0	0.34	0	0.65	0	0.99	1	0	0	0.01
SWE	1	0	1	0.79	0	0.21	1	1	0	0	0	0
SWI	0.00	0	0.32	0.20	0.59	0.75	0.91	0.95	0	0	0.09	0.05
GRB	0.00	0	0.33	0.50	0.59	0.49	0.92	0.99	0	0	0.08	0.01
Mean	0.35	0	0.51	0.47	0.30	0.51	0.81	0.98	0.15	0	0.04	0.02

Notes. The 'best-all' and the 'best-ARFIMA' posterior densities have been employed to obtain the figures.